Areally Averaged Sediment Transport Equations for Predicting Sediment from Bare Rilled Hillslopes

Gokmen Tayfur

Dept. Civil Engineering, Izmir Institute of Technology,
Urla, Izmir, Turkey
gokmentayfur@iyte.edu.tr

INTRODUCTION

- In hydrologic modelling an irregular topography is often replaced by a smooth surface,
  - because of the complications arising in the numerical procedures and
  - the extra effort involved in obtaining the microtopographic data at a grid scale needed for the numerical model.
- Tayfur et al. (1993) and Tayfur and Singh (2004) investigated surface flow and sheet sediment transport (SST) over irregular microtopographic surfaces, respectively, and found that
  - microtopograhy was one of the major factors dominating the temporal and spatial distributions of the state variables, such as flow depth, flow velocity, and sediment concentration.

- Also, in hydrologic modelling, although the importance of rills to SST has been well established experimentally both in field and laboratory studies (Emmett 1978; Abrahams and Parsons 1990; Govindaraju et al 1992, among many), they have often been neglected in numerical modelling (Govindaraju and Kavvas 1991; Parlange et al 1999; Tayfur 2001, 2002, among many).

- Yet, there have been a substantial modelling effort for sediment from rilled-microtopographic surfaces with simplified conceptualizations such as:
  - (1) Transport is treated as one-dimensional and steady
  - (2) Rills are considered to be located parallel
  - (3) Transport is considered to be occurring only in rills; etc.

(Kavvas and Govindaraju 1992, Hairsane and Ross 1992, Bulygin et al 2002).

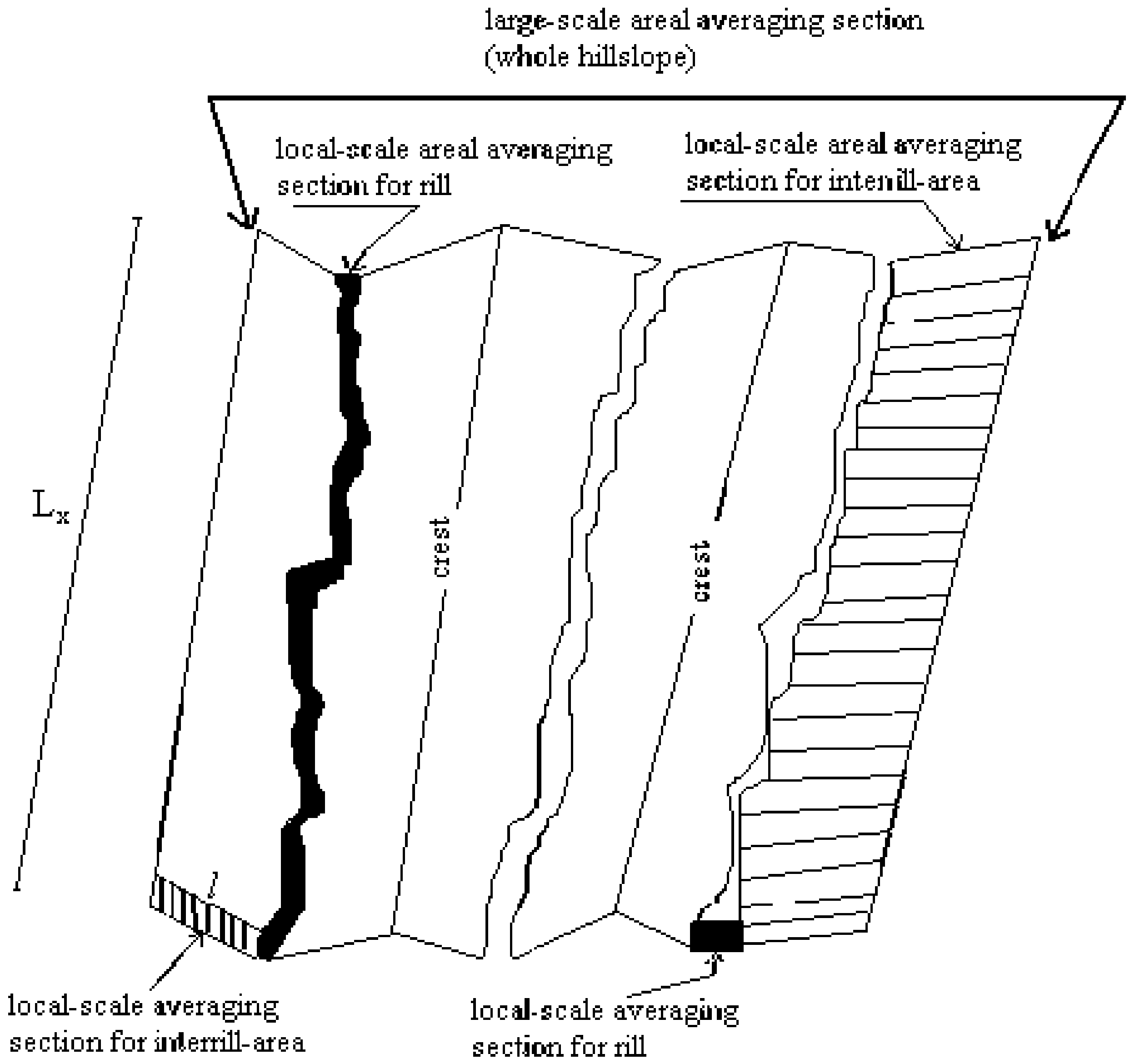
–In reality, SST
over interrill-areas occurs
mostly in two dimensions
due to the variability in
the local microtopography

there is **lateral sediment load**
from interrill areas
to adjacent rill sections.

- **The objective of this study** is to develop areally averaged equations for unsteady and non-uniform SST composed of interacting rill and interrill-area sediment transport.

ROAD MAP

- **Local-scale averaging**
- **Local-scale areal averaging**
- **Large-scale (hillslope-scale) areal averaging**


large-scale areal averaging section
(whole hillslope)
local-scale areal averaging
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- **Local-scale averaging**
  - Integrating point—scale equations over an individual interrill—area length
    - Using Liebnitz rule
    - Employing sine flow profile

- **Local-scale areal averaging**

- Integrating local—scale averaged equations over an individual interrill—area length

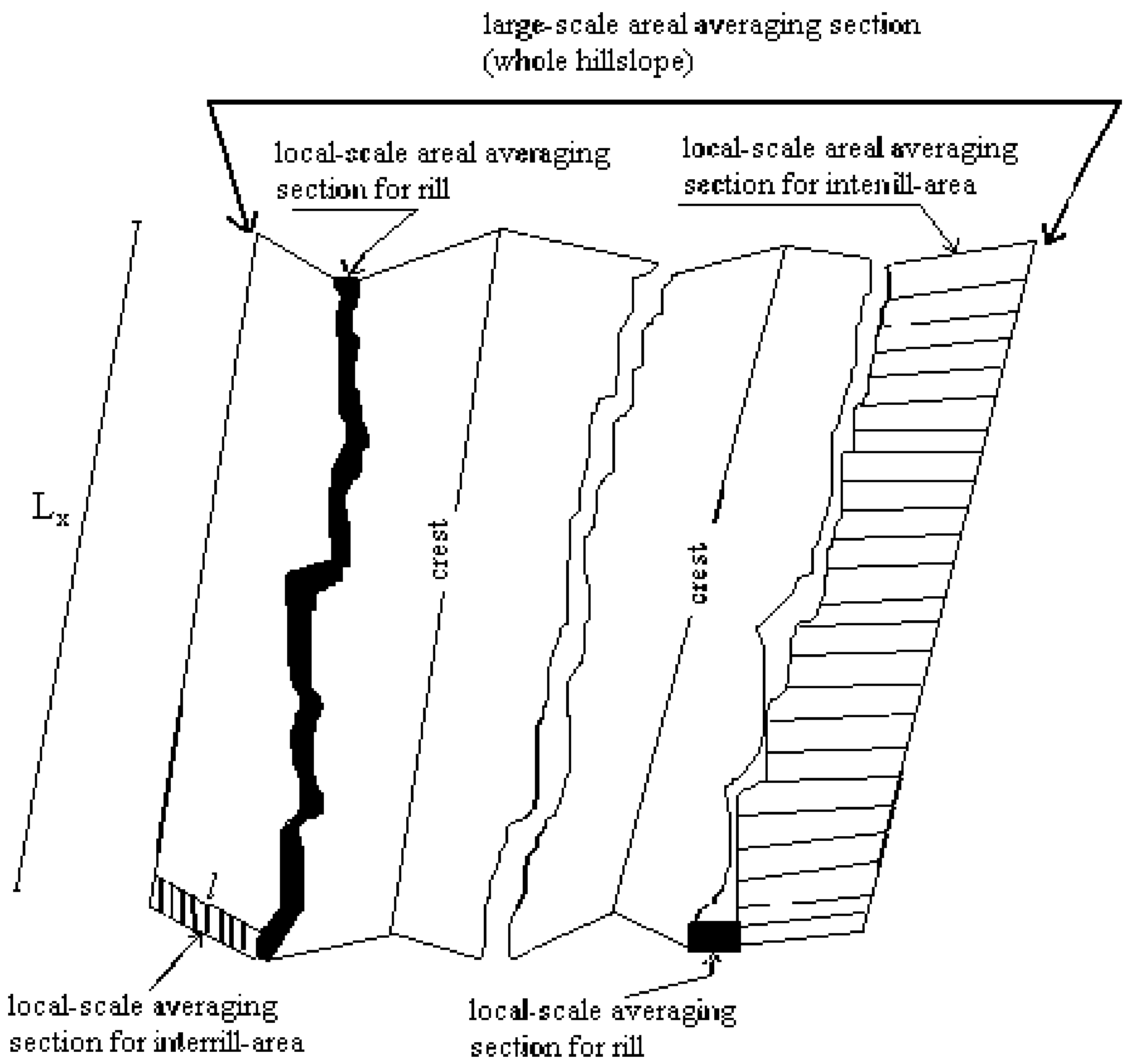
    - Using Liebnitz rule
    - Employing sine flow profile
    - Employing Taylor series

- **Large-scale (hillslope-scale) areally averaging**
  - Performing statistical averaging of the local—scale areal averaged equations. To do so;
  - employing Taylor series expansion,
  - taking into account the first two moments of the series, and
  - **assuming that all the randomness in the state variable *x* is due to the randomness in the parameters of the process.**

MATHEMATICAL MODELS

Areally Averaged Flow Equations

- **Local-scale averaging** of sheet flow and rill flow is presented in Tayfur and Kavvas (1994).
  - They averaged the 2D sheet flow equation along an interrill-area width (*l*) (Fig. 1) and also quantified the local-scale lateral flow fluxes from adjacent interrill areas to the rill section.
- Tayfur and Kavvas (1998) then performed the **local-scale areal averaging** for both the locally averaged sheet flows and rill flows.
- In the end, Tayfur and Kavvas (1998) statistically averaged the local-scale areally averaged flow equations over the whole hillslope section (see Fig. 1) to obtain the **large-scale (hillslope-scale) areally averaged** flow equations for both rill and interrill-area sections as follows: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$$\frac{\partial h_o'\left(\bar{r}'\right)}{\partial t}+0.985\sum_{i=1}^{n}\ \sum_{j=1}^{n}Cov\left(r_i,r_j\right)\left\{\frac{\partial^2\left[K_X'\left(\bar{r}'\right)h_o'^{1.5}\left(\bar{r}'\right)\right]}{\partial r_i'\,\partial r_j'}+1.97\frac{\partial^2\left[K_{yl}'\left(\bar{r}'\right)h_o'^{1.5}\left(\bar{r}'\right)\right]}{\partial r_i'\,\partial r_j'}\right\}$$

$$+1.97\left\{K_X'\left(\bar{r}'\right)h_o'^{1.5}\left(\bar{r}'\right)+1.97K_{yl}'\left(\bar{r}'\right)h_o'^{1.5}\left(\bar{r}'\right)\right\}=\left\langle q_l'\right\rangle$$

$$\frac{\partial h_r'\left(\bar{r}'\right)}{\partial t}+0.985\sum_{i=1}^{n}\ \sum_{j=1}^{n}Cov\left(r_i,r_j\right)\left\{\frac{\partial^2\left[\frac{K_R\left(\bar{r}'\right)h_r'^{1.5}\left(\bar{r}'\right)}{\left(w_{r_{Lx}}+\pi h_r'\left(\bar{r}'\right)\right)^{0.5}}\right]}{\partial r_i'\,\partial r_j'}-1.97\frac{\partial^2\left[K_{Y_i}'\left(\bar{r}'\right)\ h_o'^{1.5}\left(\bar{r}'\right)\right]}{\partial r_i'\,\partial r_j'}\right\}+$$

$$1.97\left\{\frac{K_R\left(\bar{r}'\right)\ h_r'^{1.5}\ \left(\bar{r}'\right)}{\left[w_{r_{Lx}}+\pi h_r'\ \left(\bar{r}'\right)\right]^{0.5}}-1.97h_o'^{1.5}\ \left(\bar{r}'\right)\left[K_{Y_i}'\ \left(\bar{r}'\right)\right]\right\}=\left\langle q_l'\right\rangle$$

where $h'_o(\bar{r}')$ and $h'_r(\bar{r}')$ are the large-scale areally averaged interrill-area sheet flow depth and rill flow depth at the scale of a hillslope, respectively; the vector random variable $\bar{r}$ = (C_z S_{ox}, S_{oy}, S_r, w_r, l, L_x), and $\bar{r}'$ is its hillslope-scale mean vector. q_l = net lateral flow (L/T); S_{ox} and S_{oy} = the bed slopes in x- and y-directions, respectively; l is interrill area width, L is rill length, w_r is rill width, and C_z = Chezy's roughness coefficient ($L^{1/2}/T$). K'_{yl} = expected value of K_{yl}/l over an interill area; $K'_{x_{Lx}}$ = the local-scale interrill-area average of K_x at the hillslope bottom.

- POINT—SCALE SHEET SEDIMENT TRANSPORT EQUATION

$$\frac{\partial(h_o c_o)}{\partial t} + \frac{\partial}{\partial x}\left(K_x h_o^{1.5} c_o\right) + \frac{\partial}{\partial y}\left(K_y h_o^{1.5} c_o\right) = \frac{1}{\rho_s}\left(D_{do} + D_{fo}\right)$$

$$D_{do} \quad = \quad \alpha \; r^{\beta}\left(1 - \frac{h_o + l_d}{6.69 r^{0.182}}\right)$$

$$D_{fo} \quad = \quad \varphi \; \left[\eta\left(\tau_o - \tau_c\right)^k - \rho_s c_o q_o\right]$$

c_O = sediment concentration by volume on an interrill-area
ρ_s = sediment particle density

D_{do} = soil detachment rate by raindrops in interrill-area
D_{fo} = soil detachment/deposition rate by sheet flow in interrill-area

α = soil detachability coefficient
r = rainfall intensity
l_d = loose soil depth
φ = transfer rate coefficient
η = soil erodibility coefficient;

$\tau_o =$ interrill-area shear stress

g = specific weight of water

d = particle diameter

$q_o =$ unit flow discharge in the flow direction

Local-Scale Areally Averaged Interrill-Area Sheet Sediment Transport

- Tayfur (2007) obtained the local-scale areally averaged interrill-area SST equation in two stages:
  - i. Averaging the point-scale interrill-area SST over interrill-area width (*l*) (Fig. 1) (**local-scale averaging).**
    - The point scale SST is integrated along the y-direction over an individual interrill-area width (*l*) (Fig. 1).

Tayfur (2007) obtained the following **local-scale averaged SST equation:**

$$\frac{\partial\left\langle h_o c_o\right\rangle}{\partial t}+\frac{\partial}{\partial x}\left\langle K_x h_o^{1.5} c_o\right\rangle=\left\langle\frac{1}{\rho_s}\left(D_{do}+D_{fo}\right)\right\rangle-\frac{1}{l}\left[K_y h_o^{1.5} c_o\right]_{y=l}$$

Mathematically, $\frac{1}{l}\int_0^l h_o c_o \, dy = \langle h_o c_o \rangle$. The term $\langle h_o c_o \rangle$, when expanded as a Taylor series around local-scale-averaged sheet flow depth $\langle h_o \rangle$ and local-scale-averaged sediment concentration $\langle c_o \rangle$ reduces to the zeroth order: $\langle h_o c_o \rangle = \langle h_o \rangle \langle c_o \rangle$. Using a different notation in local-scale averaging as $\langle h_o \rangle \langle c_o \rangle = \bar{h}_o \bar{c}_o$:

$$\frac{\partial\left(\bar{h}_o \bar{c}_o\right)}{\partial t}+\frac{\partial}{\partial \ x}\left(K'_x \bar{h}_o^{1.5} \bar{c}_o\right)=\frac{1}{\rho_s}\left(\bar{D}_{do}+\bar{D}_{fo}\right)-\frac{K_{yl}}{l} h_{ol}^{1.5} c_{ol}$$

where $\bar{c}_o$ = the local-scale averaged sediment concentration (L^3/L^3) on interrill area; h_{ol} = the flow depth at the outlet section of interrill area, right adjacent to the rill section (L); c_{ol} = the sediment concentration at the outlet section of interrill area, right adjacent to the rill section (L^3/L^3); $\overline{D}_{do}$ = the local-scale averaged soil detachment rate due to raindrops on interrill-area ($M/L^2/T$); and $\overline{D}_{fo}$ = the local-scale averaged soil detachment/deposition rate by sheet flow on interrill-area ($M/L^2/T$).

There are 4 unknown variables in Eq. (4)— $\bar{h}_o$, $\bar{c}_o$, h_{ol}, and c_{ol}. Therefore, in order to close the system, there is a need to develop a relation between the variables at the outlet section (h_{ol}, and c_{ol}) and the corresponding local-scale averaged variables ($\bar{h}_o$ and $\bar{c}_o$). Tayfur and Kavvas (1994), based upon an assumption of sine flow profile for flow over an interrill area, found out that i.e., $h_{ol} = 1.57\bar{h}_o$. Based upon the numerical tests, Tayfur (2007) proposed $c_{ol} = 1.5\bar{c}_o$. Thus, Tayfur (2007) obtained the local-scale-averaged interrill-area SST equation as:

$$\frac{\partial(\bar{h}_o\bar{c}_o)}{\partial t} + \frac{\partial}{\partial x}\left(K'_x \bar{h}_o^{1.5}\bar{c}_o\right) = \frac{1}{\rho_s}\left(\overline{D}_{do} + \overline{D}_{fo}\right) - 2.95\frac{K_{yl}}{l}\bar{h}_o^{1.5}\bar{c}_o$$

- ii. Averaging the resulting local-scale averaged interrill-area SST equation along the interrill-area length (*Lx*) (Fig. 1) (**local-scale areal averaging).**

Eq. (5) is quasi-two dimensional. It is in 1D, yet it contains the 2D properties of the sheet SST. In the numerical solution of this equation, one requires the average values of the model parameters over the local-scale interrill area (Fig. 1) at each point along the x-direction (along the hillslope length). Such a solution is not attractive from the data collection and computation point of view. Therefore, in order to avoid such a problem and to obtain an areally averaged conservation equation that is still local scale but covers the length of a hillslope in the x-direction toward the stream, one has to average Eq. (5) along the hillslope length (L_x) (Fig. 1). The local scale areal averaging is performed as:

$$\frac{1}{L_x}\int_0^{L_x}\frac{\partial\left(\overline{h}_o\overline{c}_o\right)}{\partial t}dx+\frac{1}{L_x}\int_0^{L_x}\frac{\partial}{\partial x}\left(K'_x\overline{h}_o^{1.5}\overline{c}_o\right)dx=\frac{1}{L_x}\int_0^{L_x}\frac{1}{\rho_s}\left(\overline{D}_{do}+\overline{D}_{fo}\right)dx-\frac{2.95}{L_x}\int_0^{L_x}\frac{K_{yl}}{l}\overline{h}_o^{1.5}\overline{c}_o dx$$

Integration of Eq. (6) yields:

$$\frac{\partial\left(h'_o c'_o\right)}{\partial t}+\frac{1}{L_x}\left(K'_{xL_x}\overline{h}_{oL_x}^{1.5}\overline{c}_{oL_x}\right)=\frac{1}{\rho_s}\left(D'_{do}+D'_{fo}\right)-2.95K'_{yl}h'^{1.5}_o c'_o$$

where c_o' = the local-scale areally averaged interrill-area sediment concentration (L^3/L^3); $\bar{h}_{oL_x}$ = the local-scale averaged flow depth at the hillslope bottom (L); $\bar{c}_{oL_x}$ = local-scale averaged sediment concentration at the hillslope bottom (L^3/L^3); D_{do}' = the local-scale areally averaged soil detachment rate due to raindrops over an interrill-area ($M/L^2/T$); and D_{fo}' = the local-scale areally averaged soil detachment/deposition rate by sheet flow over an interrill-area ($M/L^2/T$).

In developing Equ (7), $\frac{1}{L_x}\int_0^{L_x} \bar{h}_o \bar{c}_o \, dx = \left\langle \bar{h}_o \bar{c}_o \right\rangle$. The term $\left\langle \bar{h}_o \bar{c}_o \right\rangle$, when expanded as a Taylor series around local-scale areally averaged sheet flow depth $\left\langle \bar{h}_o \right\rangle$ and local-scale areally averaged sediment concentration $\left\langle \bar{c}_o \right\rangle$, reduces to the zeroth order: $\left\langle \bar{h}_o \bar{c}_o \right\rangle = \left\langle \bar{h}_o \right\rangle \left\langle \bar{c}_o \right\rangle$ and using different notation as $\left\langle \bar{h}_o \right\rangle \left\langle \bar{c}_o \right\rangle = h_o' c_o'$.

There are 4 unknown variables in Eq. (7)—h'_o, c'_o, $\bar{h}_{oLx}$, $\bar{c}_{oL_x}$ and there is a need to close the system by developing relations between the variables at the hillslope bottom ($\bar{h}_{oLx}$ and $\bar{c}_{oL_x}$) and the corresponding local-scale areally averaged variables (h'_o and c'_o). Tayfur and Kavvas (1998) assumed a sine flow profile over an interrill area [i.e., $\bar{h}_o(x,t) = h_{oL_x}(t)\sin(\frac{\pi x}{2L_x})$] which resulted in the flow depth at the bottom of the section $(\bar{h}_{oL_x})$ to be 1.57 times the local-scale-averaged flow depth on that section (h'_o); i.e., $\bar{h}_{oL_x} = 1.57h'_o$ (or $\bar{h}_{oL_x} = \frac{\pi}{2}h'_o$). Also, upon numerical tests, Tayfur (2007) came up with $\bar{c}_{oL_x} = 1.5c'_o$.

Local-scale areally averaged interrill-area SST equation is obtained as:

$$\frac{\partial(h'_o c'_o)}{\partial t} + \frac{2.95}{L_x}\left(K'_{x_{Lx}} h'^{1.5}_o c'_o\right) = \frac{1}{\rho_s}\left(D'_{do} + D'_{fo}\right) - 2.95K'_{yl}h'^{1.5}_o c'_o$$

This equation requires only the average values of the model parameters over the whole individual interrill-area.

Large-Scale Areal Averaging of Interrill-Area SST Equation

- Since there may be many individual interrill-areas over a whole hillslope, it is not feasible to solve SST for each interrill area.

- Therefore, it is desirable to average the local-scale areally averaged SST Equ (8) over the whole hillslope.

- The hillslope-scale averaging is accomplished by the statistical averaging of Equ (8) over the whole hillslope (Fig. 1):


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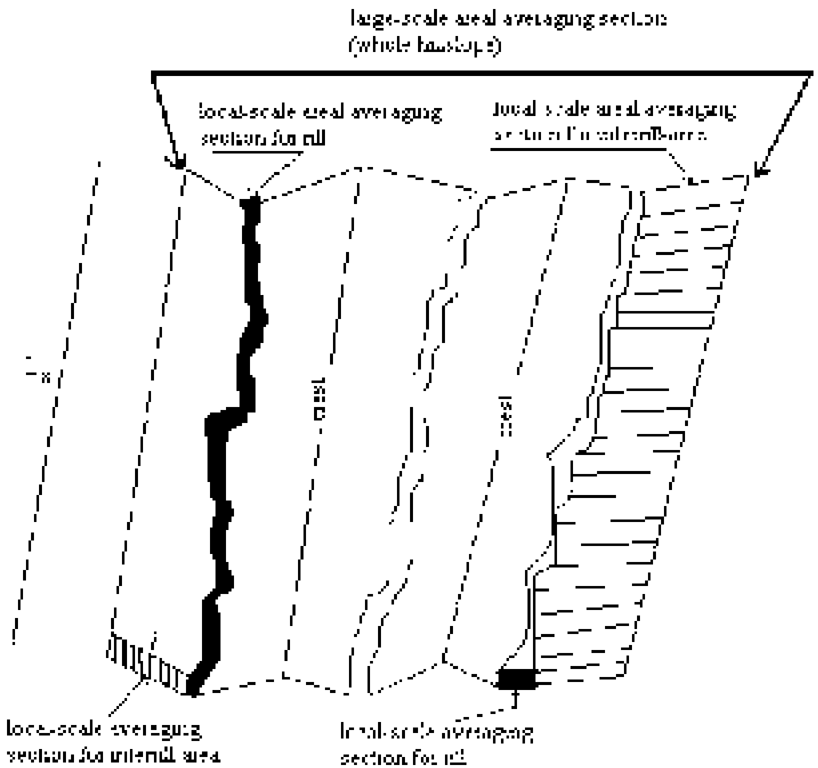
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section for rill
local-scale averaging
section for interrill area
local-scale averaging
section for rill

$$\frac{\partial\left\langle h_o' c_o' \right\rangle}{\partial t} + 2.95\left\langle K_X' h_o'^{1.5} c_o' \right\rangle = \frac{1}{\rho_s}\left(\left\langle D_{do}' \right\rangle + \left\langle D_{fo}' \right\rangle\right) - 2.95\left\langle K_{yl}' h_o'^{1.5} c_o' \right\rangle$$

where $\left\langle \quad \right\rangle$

stands for the statistical average (expectation) value of a variable over the whole hillslope.

To obtain an explicit expression from Equ (9) in terms of statistical averages of its individual terms, it is necessary to find the expectations of the product terms containing more than one variable in Equ (9).

These expectations may be found using the Taylor series.

The Taylor series expansion of a function $f(x, \bar{r})$

around $\bar{r} = \bar{r}'$ can be expressed to the second order as:

$$f(x,\bar{r}) = f(x,\bar{r}') + \sum_{i=1}^{n} (r_i - r_i') \left. \frac{\partial f(x,\bar{r})}{\partial r_i} \right|_{\bar{r}=\bar{r}'} + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \left. \frac{\partial^2 f(x,\bar{r})}{\partial r_i \partial r_j} \right|_{\bar{r}=\bar{r}'} (r_i - r_i')(r_j - r_j')$$

In Equ (10), ***x*** represents the state variable;

$\bar{r}$ represents a set of random parameters, such as slope, roughness coefficient and interrill area width, and is a random vector.

$\bar{r}'$ is the mean value of this random vector.

It is assumed that all the randomness in the state variable *x* is due to the randomness in the parameters of the process.

Such an assumption is plausible, since the dynamics is controlled by model parameters. Therefore, any randomness that may occur in the state variable would be due to the randomness that occurs in model parameters. Under this assumption;

$$\left\langle f\left(x,\bar{r}\right)\right\rangle = f\left(x,\bar{r}'\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2 f(x,\ \bar{r}')}{\partial r_i'\,\partial r_j'}\,Cov\left(r_i\ ,r_j\right)$$

Applying Equ (11) to Equ (10),

$$\left\langle K'_X h_o'^{1.5} c'_o \right\rangle = K'_X\left(\bar{r}'\right) h_o'^{1.5}\left(\bar{r}'\right) c'_o\left(\bar{r}'\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\left[K'_X\left(\bar{r}'\right) h_o'^{1.5}\left(\bar{r}'\right) c'_o\left(\bar{r}'\right)\right]}{\partial r'_i\,\partial r'_j} Cov\left(r_i, r_j\right)$$

$$\left\langle K'_{yl} h_o'^{1.5} c'_o \right\rangle = K'_{yl}\left(\bar{r}'\right) h_o'^{1.5}\left(\bar{r}'\right) c'_o\left(\bar{r}'\right) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\partial^2\left[K'_{yl}\left(\bar{r}'\right) h_o'^{1.5}\left(\bar{r}'\right) c'_o\left(\bar{r}'\right)\right]}{\partial r'_i\,\partial r'_j} Cov\left(r_i, r_j\right)$$

where the vector random variable $\bar{r}$ = $(C_z, S_{ox}, S_{oy}, L_x, l)$ and

$\bar{r}'$ is its hillslope-scale mean vector.

Substituting Eqs (13) and (12) into Eq. (10),
upon arrangement,

the large-scale (hillslope-scale) areal averaged interrill-area SST equation is obtained as follows:

$$\frac{\partial\left(h'_o(\bar{r}')c'_o(\bar{r}')\right)}{\partial\ t}+1.48\sum_{i=1}^{n}\sum_{j=1}^{n}Cov\left(r_i,r_j\right)\left\{\frac{\partial^2\left[K'_X(\bar{r}')h'^{1.5}_o(\bar{r}')c'_o(\bar{r}')\right]}{\partial r'_i\ \partial r'_j}+\frac{\partial\left[K'_{yl}(\bar{r}')h'^{1.5}_o\ (\bar{r}')c'_c(\bar{r}')\right]}{\partial\ r'_i\ \partial\ r'_j}\right\}$$

$$+2.95\left\{K'_X(\bar{r}')h'^{1.5}_o(\bar{r}')c'_o(\bar{r}')+K'_{yl}(\bar{r}')h'^{1.5}_o(\bar{r}')c'_o(\bar{r}')\right\}=\frac{1}{\rho_s}\left[D'_{do}(\bar{r}')+D'_{fo}(\bar{r}')\right]$$

where

$\bar{r}$

is the vector random variable and

$\bar{r}'$

is its hillslope-scale mean vector;

$h'_o\left(\bar{r}'\right)$

= the hillslope-scale-averaged interrill-area sheet flow depth;

$c'_o\left(\bar{r}'\right)$

= the hillslope-scale-averaged interrill-area sediment concentration;

$D'_{do}\left(\bar{r}'\right)$

= the hillslope-scale-averaged soil detachment rate by raindrops over an interrill-area; and

$D'_{fo}\left(\bar{r}'\right)$

= the hillslope-scale-averaged soil detachment/deposition rate by sheet flow over an interrill-area.

Large-Scale Areal Averaging of Rill Sediment Transport Equation

- The hillslope-scale averaging is accomplished by the statistical averaging of the following equation over the whole hillslope (Fig. 3) (Tayfur 2007).

$$\frac{\partial\left(h'_r c'_r\right)}{\partial t}+\left[\frac{2.95 K_R {h'_r}^{1.5} c'_r}{\left(w_{r_{Lx}}+\pi h'_r\right)^{0.5}}\right]=\frac{1}{\rho_s} D'_{fr}+2.95\, {h'_o}^{1.5} c'_o K'_{Y_i}$$

- The hillslope scale averaging is accomplished in a similar fashion as it is briefly presented above for the interrill area SST equation.

- Hence, for the sake of brevity, the final version of the derived hillslope-scale averaged rill sediment transport equation is obtained as follows:

$$\frac{\partial\left(h_r'\left(\bar{r}'\right)c_r'\left(\bar{r}'\right)\right)}{\partial t}+1.48\sum_{i=1}^{n}\sum_{j=1}^{n}Cov\left(r_i,r_j\right)\left\{\frac{\partial^2\left[\frac{K_R\left(\bar{r}'\right)h_r'^{1.5}\left(\bar{r}'\right)c_r'\left(\bar{r}'\right)}{\left(w_{r_{Lx}}+\pi h_r'\left(\bar{r}'\right)\right)^{0.5}}\right]}{\partial r_i'\ \partial r_j'}-\right.$$

$$\left.\frac{\partial^2\left[K_{Y_1}'\left(\bar{r}'\right)h_o'^{1.5}\left(\bar{r}'\right)c_r'\left(\bar{r}'\right)\right]}{\partial r_i'\ \partial r_j'}-\frac{\partial^2\left[K_{Y_2}'\left(\bar{r}'\right)h_o'^{1.5}\left(\bar{r}'\right)c_r'\left(\bar{r}'\right)\right]}{\partial r_i'\ \partial r_j'}\right\}+$$

$$2.95\left\{\frac{K_R\left(\bar{r}'\right)h_r'^{1.5}\left(\bar{r}'\right)c_r'\left(\bar{r}'\right)}{\left[w_{r_{Lx}}+\pi h_r'\left(\bar{r}'\right)\right]^{0.5}}-h_o'^{1.5}\left(\bar{r}'\right)c_r'\left(\bar{r}'\right)\left[K_{Y_1}'\left(\bar{r}'\right)+K_{Y_2}'\left(\bar{r}'\right)\right]\right\}=\frac{1}{\rho_s}D_{fr}'\left(\bar{r}'\right)$$

where

$\bar{r}$

is the vector random variable and

$\bar{r}'$

is its hillslope-scale mean vector;

$h_r'\left(\bar{r}'\right)$

= the hillslope-scale-averaged rill flow depth computed by Eq. (9);

$c_r'\left(\bar{r}'\right)$

= the hillslope-scale-averaged rill sediment concentration; and

$D_{fr}'\left(\bar{r}'\right)$

= the hillslope-scale-averaged rill soil detachment/deposition rate.

- To obtain the complete solution for sediment transport at the scale of a hillslope;

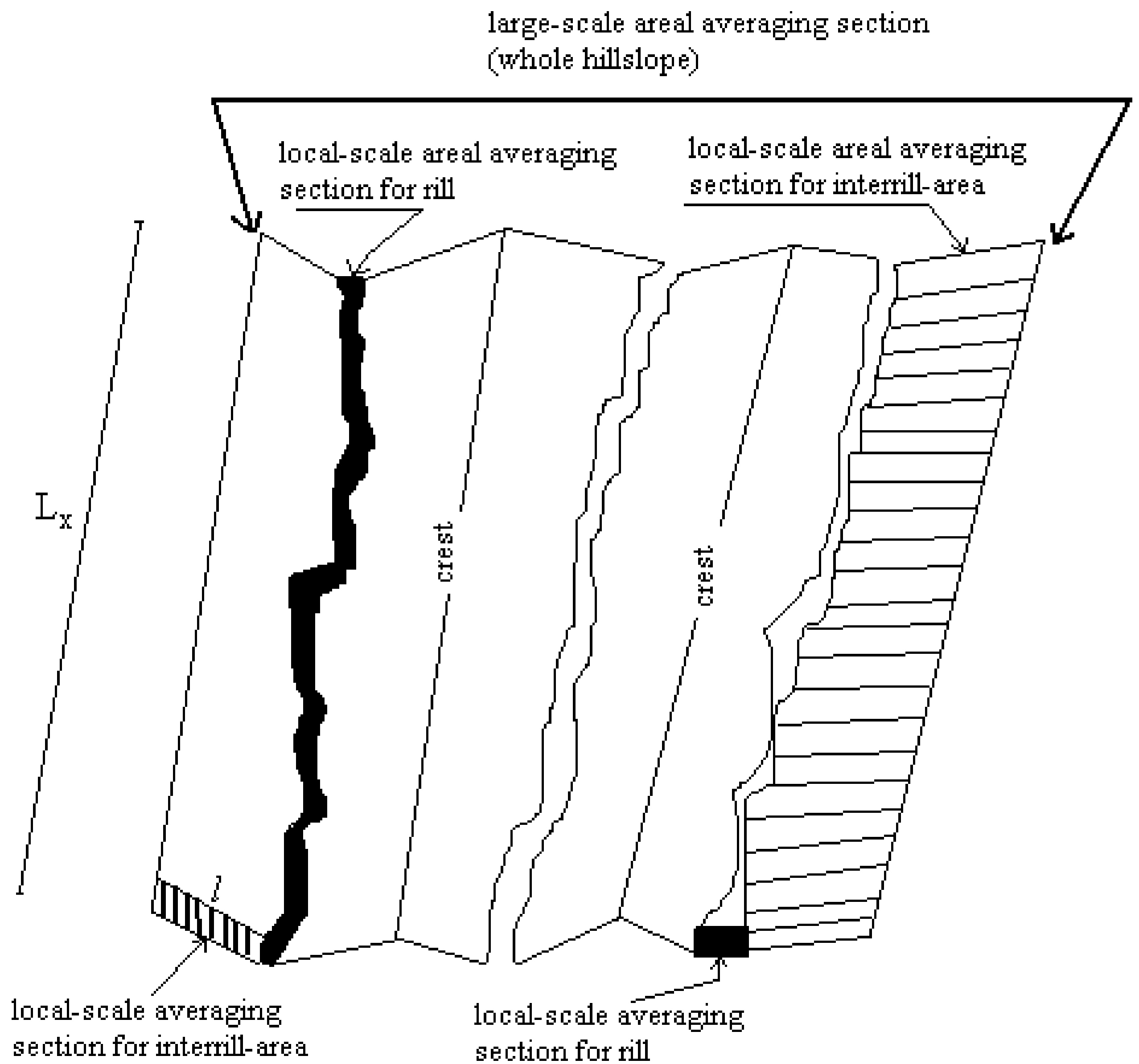
- the **hillslope-scale averaged** interrill-area **sheet flow** Equ (1) and hillslope-scale averaged **rill flow** Equ (2) are first solved simultaneously

  – to obtain the hillslope-scale averaged flow depths and fluxes over interrill-areas and in rill sections that are the required inputs in the solution of the hillslope-scale sediment transport equations (14) and (16).

- Then, **hillslope-scale sediment transport equations** (14) and (16) are solved simultaneously for each time step.

  - Equ (14) is first solved to calculate the hillslope-scale averaged sediment discharge going to the neighbouring rill section and coming to the stream located at the downstream end of the interrill-area section.

  - Then, Equ (16) is solved to calculate the hillslope-scale averaged sediment discharge coming from the rill section to the stream located at the end of the hillslope bottom.



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In order to determine the **total sediment discharge from a hillslope** to a neighbouring stream, **the number of rills over a hillslope is estimated first.**

- The **probability of the rill occurrence λ is then estimated** for the whole hillslope.
  - The hillslope-scale averaged **rill sediment discharge is multiplied by λ**, and the hillslope-scale averaged **interrill-area sediment discharge is multiplied by (1- λ)** in order to weigh the relative contributions at the scale of a hillslope.
  - **These products are then summed up to find the total sediment discharge** from a hillslope.

MODEL TESTING

- The mathematical model was tested against experimental data obtained from rainfall simulation over a cut bare hillslope located at Buckhorn Summit in Northern California, USA.

- The experimental hillslope has a steep slope of about 67%.

- Experiments were carried out in September 1990.

- The lower portion of the hillslope, had dimensions of

  - 15 m long and
  - 10 m wide,

  - it was subjected to intense rainfall intensity of 152 mm/h for duration of 10 minutes.

- The sediment discharge was measured using a Parshall flume.

- The rill structure measurements were made using a tape measure and a ruler at 11 locations along the slope spaced at 1.5 m intervals.

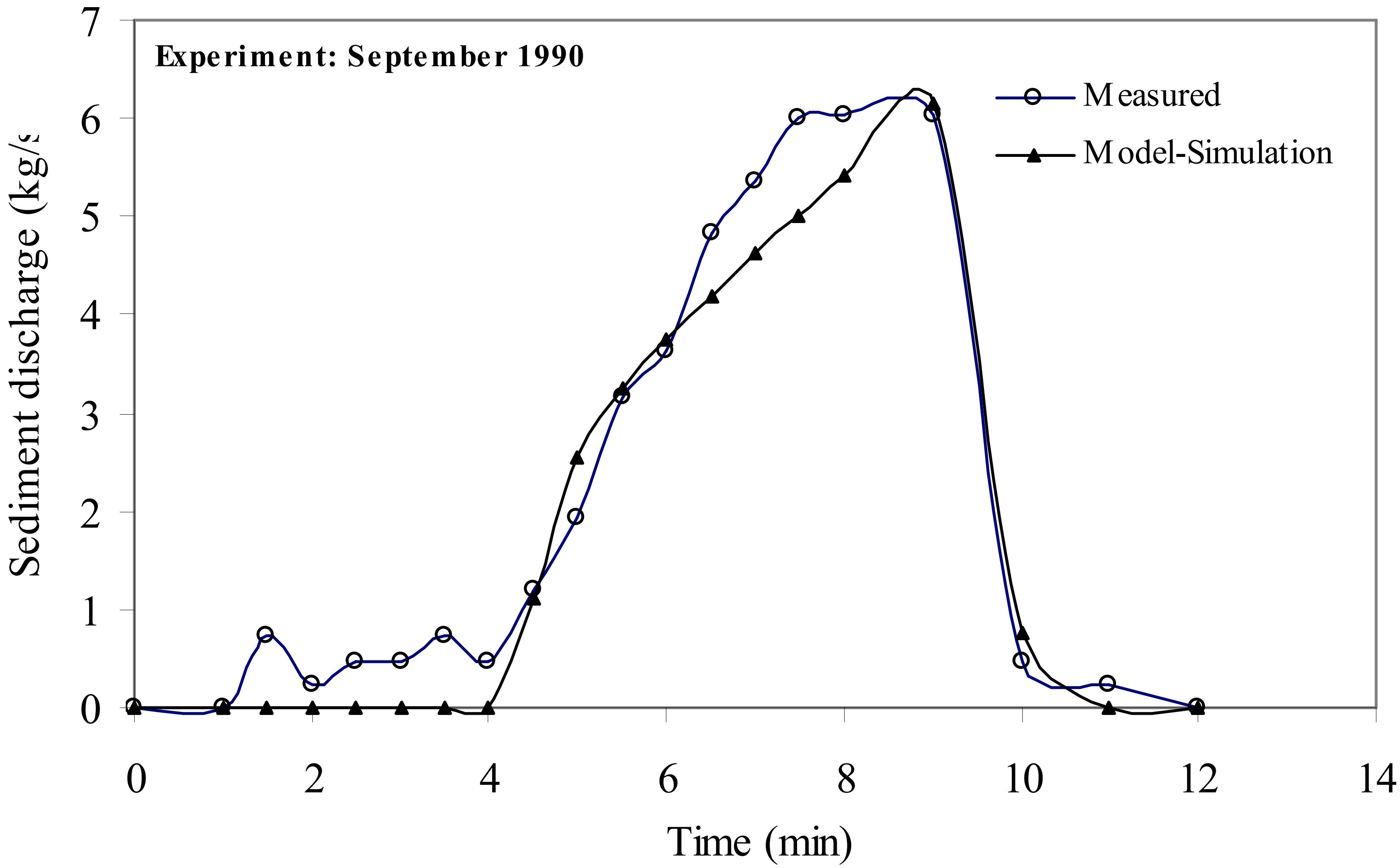
- Table 1 summarizes the measured rill depth, rill width, and expected spatial rill density.

- For this hillslope Cz = 16.6 $m^{0.5}/s$, Ks = 37.8 mm/h, k = 0.0014 1/s (rate constant), fo = 127 mm/h ρs = 2662 kg/m^3 (Govindaraju et al 1992).

Table 1 Summary of rill information masured at Buckhorn Summit Experimental Hillslope (Govindaraju et al 1992) (ESRD: Expected spatial rill density)

Distance (m)	**ESRD** (%)	**Rill Depth** (cm)	**Rill Width** (cm)
10	15%	6	10
12.5	20%	9	15
15	28%	10	22
17.5	33%	12	23
20	36%	13	24
22.5	38%	13	24
MEAN	**28.3%**	**10.5**	**19.7**

- Fig.2 shows simulation of measured sediment discharge data by the areal averaged sediment transport equations.


Experiment: September 1990
Measured
Model-Simulation
Sediment discharge (kg/s
Time (min)
0
1
2
3
4
5
6
7
8
10
12
14

CONCLUDING REMARKS

- One requires local roughness and local *x*- and *y*-direction slope values at every nodal point of a computational network mesh over a hillslope when modelling two-dimensional SST by the point-scale equations.

- This results in a very substantial parameter estimation problem.

- On the other hand, when one models SST by the hillslope-scale averaged equations, one requires only the averaged values of one roughness coefficient, one x-direction slope and one y-direction slope over the whole interrill area, and one x-direction bed slope for all rills over the whole hillslope section.

- For example; a hillslope has typical dimensions in the range of 120—600 m in the mass transport direction and in the range of 120—1200 m in the longitudinal direction.

- If a hydrologist is provided with only 30 m x 30 m resolution data, he/she would have in the range of 16—800 local-scale sediment transport parameter sample values that could be utilized to estimate the mean and the covariance *Cov (ri, rj)* parameters which appear in equations (14) and (16).

- Although the hillslope-scale areally averaged model uses **significantly less information on the land-surface microtopography**, it may perform as well as does the point-scale model.

References

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